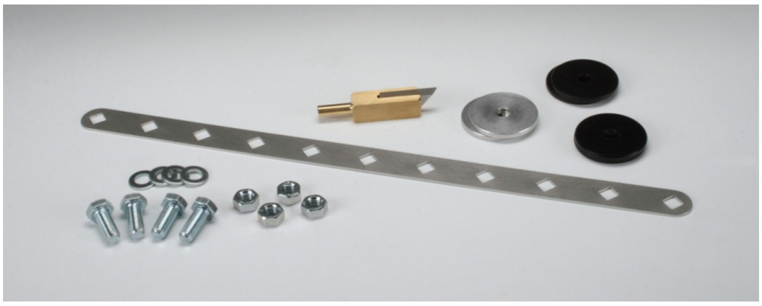
Physical pendulum

Number	135610-EN	*Topic*	Mechanics, rigid bodies				
Version	2016.08.11 / HS	*Type*	Student exercise	*Suggested for*	grade 12+	*p.*	1/5



Objective

To study the physical pendulum – i.e. a composite, rigid body – comparing measured and calculated values of moments of inertia.

Principle

The moment of inertia is determined by measuring the period for the physical pendulum.
The emphasis is on symmetric mass distributions which places the pendulum's centre of mass in the centre of the pendulum. This simplifies the calculations.

The experiment can be extended to arbitrary mass distributions.

Equipment

(See Detailed List of Equipment on the last page)

The physical pendulum 218100 consists of a steel rod with a row of holes which are used both as pivots and for attaching weights. The fixed part of the bearing is formed by a knife edge which is attached to stand material or even better clamped to a table edge.

The equipment is provides with 4 disks of steel and 2 of aluminium. These are used in pairs – placed on each side of the steel rod with a bolt. Hereafter, two disks, a bolt and a nut is termed a *weight*.

The centre of mass, the pivot and the moment of inertia can be varied in countless ways. The calculations make good use of the parallel axis theorem and a range of formulas for inertial moments of different parts of the pendulum. These are found in the *Moments of inertia* section. Practical calculations are best performed in a spread sheet.

(Two extra sets of nuts and bolts are provided to use as "trimming weights" for use in connection with the Bessel Pendulum – see experiment 135630-EN. The Bessel Pendulum also uses four washers with the large weights.)

Measuring the Period of the Pendulum

Select one of the four different methods given below.

With a stopwatch

You must measure the time for a number of *complete* swings and divide by the number. Precision increases if the watch is started and stopped when the pendulum passes its lowest point where its speed is highest. Use a fixed point behind the pendulum as a reference and don't move your head between start and stop.

Realistically, you cannot hope for uncertainties less than 0.2 s.
If you aim for a 0.5 % precision, the total measuring time must therefore be at least 40 s.

With data logging

Place a motion sensor close by the pendulum, preferably pointing at a weight. (It takes some luck to hit the narrow rod, but it can be done.) Adjust the software to log *position* with a sample rate of 100 Hz. Check that data follows a sine curve reasonably well – large spikes indicate that the sensor misses the target.

Measure for "sufficiently long time" – about a minute.

Fit the data with a *damped harmonic oscillation*. Make sure that the fit parameters are shown with sufficient number of digits.

Depending on the software, you get the period T directly, alternatively $\omega = 2\pi/T$.

With a photogate and a timer

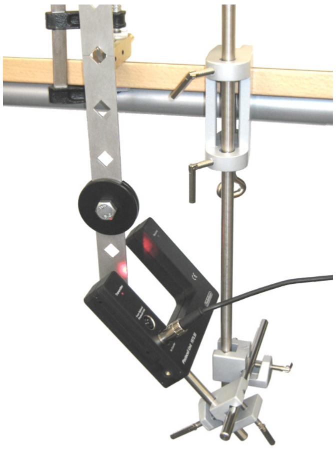
Let the pendulum hang motionless. Position the photogate so that the light ray "touches" the edge of the rod – see photo.
Photogate 197550 has a green LED that goes off when the light ray is blocked.

With the pendulum swinging (small amplitude!) the light ray must be blocked for a complete half period and pass through for the other half. This way a period is exactly the time from one blocking to the next.

With **timer 200250** the procedure is like this:

Plug the photogate into DIN socket A.

1. Pull the pendulum a little away from the light ray during the following points
2. Press *Select* until the lamp next to *Period* turns on
3. Wait until the lamp *Continuous* turns on, then press *Memory/Continuous*
4. At last, press *Start/Stop*
5. Now release the pendulum



Results are displayed as the average of two periods – write down.
Continue for sufficiently long time. Calculate the mean value.

With SpeedGate

SpeedGate (197570) has two light rays; in this experiment we use the one marked "X".

A status indicator in the display is active when the light ray is blocked. With a motionless pendulum, the "X" light ray should just graze the rod.

Use only small amplitudes: The light ray must be blocked for one half of the period. Select *Period* and *Mean Period* (not *Pendulum Period*).

1. Start the pendulum with a *small* amplitude
2. Press *Reset*
3. Read the mean period when the chosen measuring time expires.

A bit of theory

We will consider a rigid body as composed of a large number of small parts.
The **moment of inertia** of the body with respect to a given axis of rotation is then the sum of the contributions from the individual parts, each of the form

$$I_j = m_j \cdot r_j^2$$

where m_j is the mass of part no. j and r_j is the distance from this part to the axis of rotation.

The total moment of inertia is then given by the sum

$$I = \sum_j I_j$$

In practice, moments of inertia are found by integration. If the shape of the body is simple enough this can be done analytically. A number of relevant examples of this can be found in a later section.

Let the moment of inertia for a body with respect to an axis *through the centre of mass* of the body be called I_G. From this, the moment of inertia I, with respect to an arbitrary axis, parallel to the other one, can be found via the **parallel axis theorem** (aka the **Huygens–Steiner theorem**):

$$I = I_G + Ma^2$$

Here, M is the mass of the body in question and a is the distance between the two axes.

This theorem is extremely useful for calculating moments of inertia, except from the most simple cases.

The term "physical pendulum" is used when a rigid body is suspended from an axis that does *not* go through its centre of mass.

The **period** of the physical pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{Mag}}$$

where I is the moment of inertia with respect to the axis of rotation, M is the total mass, a is the distance from the centre of mass to the axis, and g is the acceleration due to gravity.

Moments of inertia

It follows from the definition that the total moment of inertia of a rigid body can be found as the sum of the moments of inertia of the individual parts of the body (with respect to the same axis).

In this specific case, we will divide the pendulum rod in a rectangular section and two semi-circular ends. (From this we must subtract the material from the 11 quadratic holes, if you want to be very precise.) The mass of the rod should be distributed among its three parts proportional to their area.

The large disks are cylindrical with a cylindrical hole in the middle. Each weight is fixed by a bolt that we will consider as a point mass. (This means that its moment of inertia with respect to its own centre of mass is 0.)

Below you find some formulas for moments of inertia.

Rectangle with dimensions b x d and mass m:

$$I_Z = \frac{m}{12} \cdot (b^2 + d^2)$$

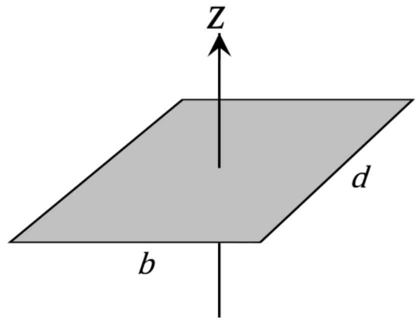
Semi-circle with radius r and mass m, displaced from the axis by a distance D:

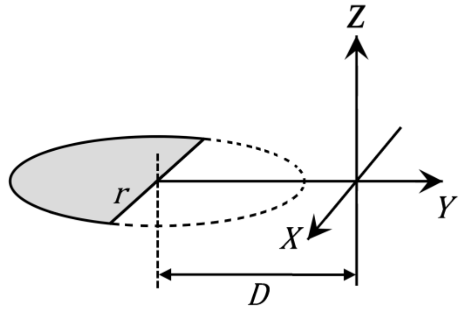
$$I_Z = m \cdot \left(D^2 + \frac{r^2}{2} + \frac{8Dr}{3\pi} \right)$$

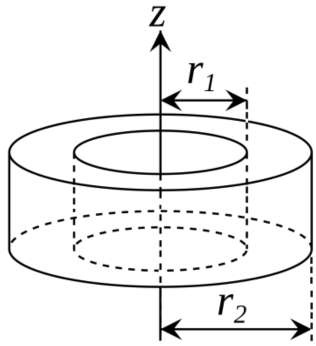
Cylinder with outer radius r_2, inner radius r_1 and mass m:

$$I_Z = \frac{m}{2} \cdot (r_1^2 + r_2^2)$$

The practical calculations on the different parts of the pendulum are best done in a spreadsheet.







Physical pendulum – Procedure

For these experiments, you don't use the washers or the extra bolts. There are still more than enough ways to combine the parts!

Experiments 1, 2 and 3 below are mutually independent.

The metal disks are fixed by the bolts – it is sufficient to tighten the nut hard with your fingers.

For simple demonstration experiments, the knife-edge bearing can be mounted in a retort stand with a heavy "A" foot or a table clamp.
More precise results are achieved by clamping the bearing directly to a table edge. Preferably directly above a table leg. And if possible, use a table that is bolted to the wall.

Note: There is a position difference when a hole is used as a pivot (upper corner), resp. is used for placing a weight (centre). Take care to use the right values in calculations.

The amplitude of the oscillations must be small. About half a centimetre is fine.

1 – Moment of inertia of the rod alone

Measure the average period over at least 20 periods for each of the 5 possible pivots (the middle hole is not used here).

Measure the distances from the centre of the rod to each of the pivots meticulously. The centre is marked by a thin line. It may be useful temporarily to extend this line across the centre hole with a strip of adhesive tape. (These distances are also used in the following experiments.)

2 – Symmetrical mass distribution

Chose a position of the iron (black) weights – e.g. hole no. 2 from the ends. The positions must be symmetrical, thus keeping the centre of mass identical to the centre of the rod.
To keep the pendulum from "capsizing" when using the centre hole, let the two bolts point in opposite directions

Measure the period as an average over at least 20 periods for each of the 5 possible pivots (including the middle one).

If you don't already have these values: Measure the distances from the centre of the rod to each of the pivots meticulously. (See ***1*** above.)

Repeat eventually with the weights in a new – but still symmetrical – position.

3 – Arbitrary mass distribution

We will now no longer demand that the mass distribution is symmetrical. We will therefore have to determine not only the moment of inertia of the pendulum but also its centre of mass.

If you don't already have these values: Measure the distances from the centre of the rod to each of the pivots meticulously. (See ***1*** above.)

In order to specify positions unambiguously, we define a coordinate axis along the rod with origin at the centre and positive direction upwards. Positions below the centre of the rod are negative.

Name the position of the pivot x_O. The positions of the weights are termed x_A resp. x_B and their masses m_A resp. m_B.

Carefully write down the experimental conditions and measure the periods as averages over at least 20 oscillations.

Calculations

1 – Moment of inertia of the rod alone

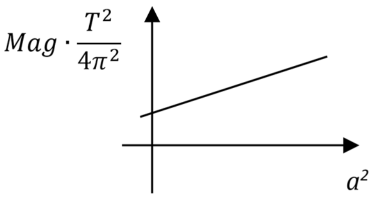
The formula for the period is re-written as

$$Mag \cdot \frac{T^2}{4\pi^2} = I$$

As the centre of mass is at the centre of the rod, all values on the left side are known. The total mass M is simply the mass of the rod m_R. For the rod alone, the right side can be re-written using the parallel axis theorem:

$$I = I_R + m_R \cdot a^2$$

This means that if the quantity $Mag \cdot \frac{T^2}{4\pi^2}$ is plotted as a function of a^2 the results will lie on a straight line with the mass of the rod m_R as the slope and the moment of inertia I_R (with respect to the centre of the rod) as the intersection with the y axis.



For comparison, the moment of inertia of the rod can be calculated by dividing it into a rectangular piece and two semi-circular ends – see the "Moments of inertia" section. The error of dropping a full treatment of the quadratic holes is quite small – but subtract their area when distributing the mass of the rod proportional to the areas of the three parts.

2 – Symmetrical mass distribution

The formula for the period is re-written as

$$Mag \cdot \frac{T^2}{4\pi^2} = I$$

As the weights are placed symmetrically, the centre of mass is at the centre of the rod; thus all values on the left side are known. Right side can be re-written using the parallel axis theorem:

$$I = I_G + M \cdot a^2$$

This means that if the quantity $Mag \cdot \frac{T^2}{4\pi^2}$ is plotted as a function of a^2 the results will lie on a straight line with the total mass M as the slope and the moment of inertia of the pendulum I_G (with respect to its centre of mass) as the intersection with the y axis.

This measured value of I_G can be compared with one calculated from theory:

$$I_G = I_R + 2(I_W + m_W x_W^2)$$

where I_R is the moment of inertia of the rod with respect to its centre, I_W is the moment of inertia of one weight with respect to its centre axis, m_W is the mass of a weight (incl. bolt set), and x_W is the distance between the centres of the rod and the weights.

I_R and I_W are calculated using the formulas in the section *Moments of inertia.* For I_R, see also ***1*** *– Moment of inertia of the rod alone*. The weights are treated as two cylinders (i.e. extended bodies) and a nut and bolt that can be considered as point masses.

If more than one measurement series is performed, the graphs will be parallel.

3 – Arbitrary mass distribution

As the centre of mass of the rod has coordinate 0, the position of the centre of mass x_G reduces to:

$$x_G = \frac{m_A \cdot x_A + m_B \cdot x_B}{M}$$

The distance from the pivot to the centre of mass is

$$a = x_O - x_G$$

The moment of inertia of the pendulum (with respect to the pivot) is found by adding contributions from the rod and the two weights

$$\begin{aligned} I = \; & I_R + m_R \cdot x_O^2 \\ + \; & I_A + m_A \cdot (x_O - x_A)^2 \\ + \; & I_B + m_B \cdot (x_O - x_B)^2 \end{aligned}$$

where I_A and I_B denotes moments of inertia of weight A and B with respect to their centre axes.

Now, a theoretical value for the period can be found and compared with the experimental results.

Discussion and evaluation

In all three experiments it is possible to compare experimental and theoretical values.

In this treatment of the results, it will be natural to pay attention to the experimental uncertainties of the measured quantities.

Teacher's notes

Concepts

Centre of mass
– presumed known

Moment of inertia
Parallel axis theorem
Period for physical pendulum
– formulas are given

Mathematical skills

Solving equations, trigonometrical functions, use of spreadsheet, graphs

In order to deduce the formulas of the moments of inertia, calculus (integrals) must be known

These experiments generally demand overview and systematic work

Didactic considerations

The calculation of the moment of inertia of the rod can be simplified by partially ignoring the holes. Meaning that we consider the rectangular part of the rod to be homogenous, but slightly lighter than it would be without holes.
This results in slightly more than 1 % error for the moment of inertia with respect the rod's centre. Calculated periods for the rod alone will be too large with up to 0.5 %. (Largest for the centre hole.) With the weights mounted, the deviation will be reduced.

An even more obvious simplification is to consider nuts and bolts as point masses.
The moment of inertia is then off by less than 0.08 % , the period by half of this value.

These errors can presumably be ignored compared to the measurement uncertainties and other error when working with the general physical pendulum.

(Note: When used as a reversion pendulum, all of these errors are completely irrelevant!)

It might seem tempting to expand experiment *2 – Symmetrical mass distribution* to determine the moment of inertia of the weights with respect to their symmetry axis. This requires subtraction of two almost identical quantities and will be subject to large uncertainties. It can only be recommended if you wish to treat uncertainty calculations in detail – or else the deviations from theory will only cause frustrations.
The best you can do is to repeat experiment 2 for all possible values of x_W , and then plot I_G as a function of x_W^2 .
The intersection with the y axis will then be $I_R + 2\,I_W$.

On www.frederiksen.eu you can find complete spreadsheets for determining moments of inertia etc. Search for item number 218100

Detailed equipment list

Specifically for the experiment

218100	Physical pendulum / Bessel-pendulum
001510	Clamp

Larger equipment

Option: Timing with SpeedGate

197570	SpeedGate

Option: Timing with photogate and timer

200250	Universal counter/timer
197550	Photogate

Option: Timing with a datalogger
Motion sensor
Logger or link to PC

Standard lab equipment

(Depending on the timing equipment)

001600	Table clamp
002310	Bosshead, square (1-2 are used)
000850	Retort stand rod 25 cm
000820	Retort stand rod 75 cm
000100	Retort stand Base