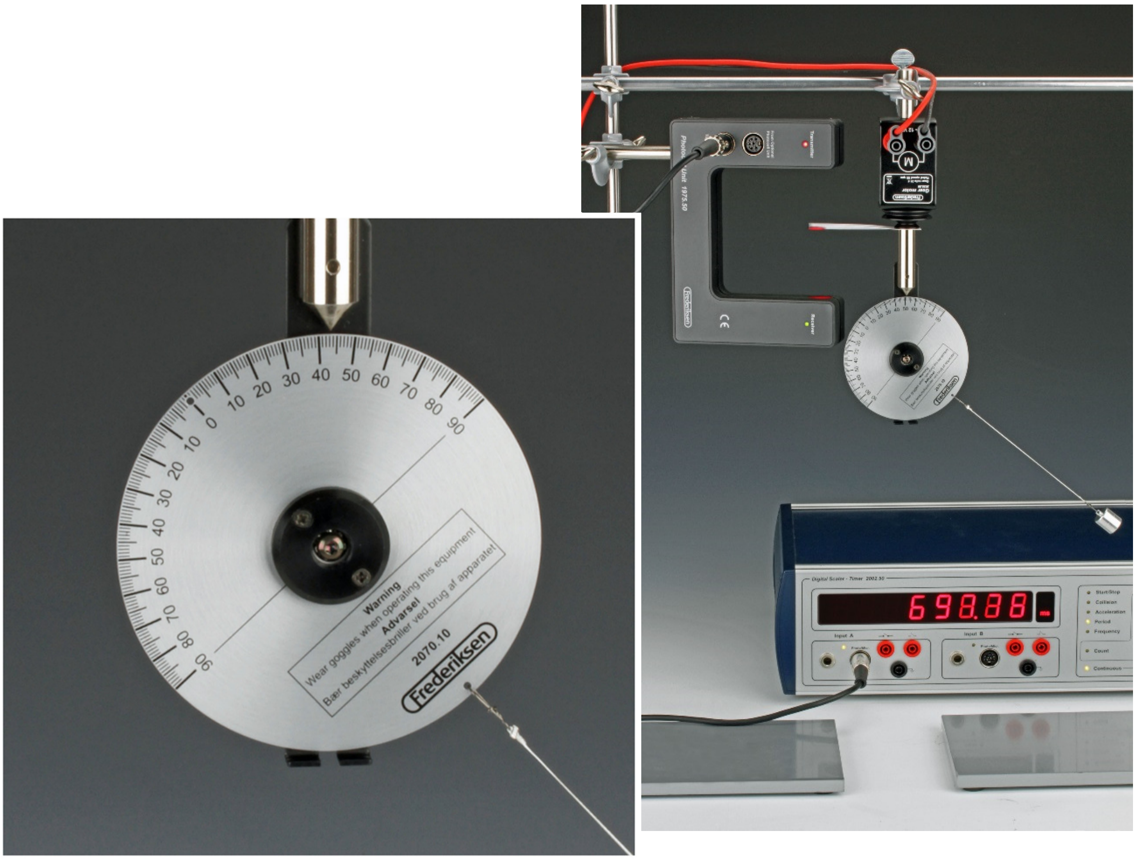
Circular motion with conical pendulum

Number	135710-EN	*Topic*	Mechanics, two-dimensional motion				
Version	2017-02-17/HS	*Type*	Student exercise	*Suggested for*	grade 11-12+	*p.*	1/4



Objective

To investigate how the centripetal force depends on orbital radius and orbital period.

Principle

We use a conical pendulum in this experiment. The bob performs a circular motion under the influence of the tension of the string and the force of gravity. The angle between these two forces is read on the fly on the graduated scale on the conical pendulum.

The orbital period can be found with a stopwatch or with a photogate.

Equipment

(See Detailed List of Equipment at the last page)

207010 Conical pendulum
202550 gear motor

DC power supply
Stand material

SpeedGate – *or*
Photogate and timer (– *or*
Stopwatch)

Frederiksen Scientific A/S
Viaduktvej 35 · DK-6870 Ølgod
Tel. +45 7524 4966
Fax +45 7524 6282
info@frederiksen.eu
www.frederiksen.eu

Setup

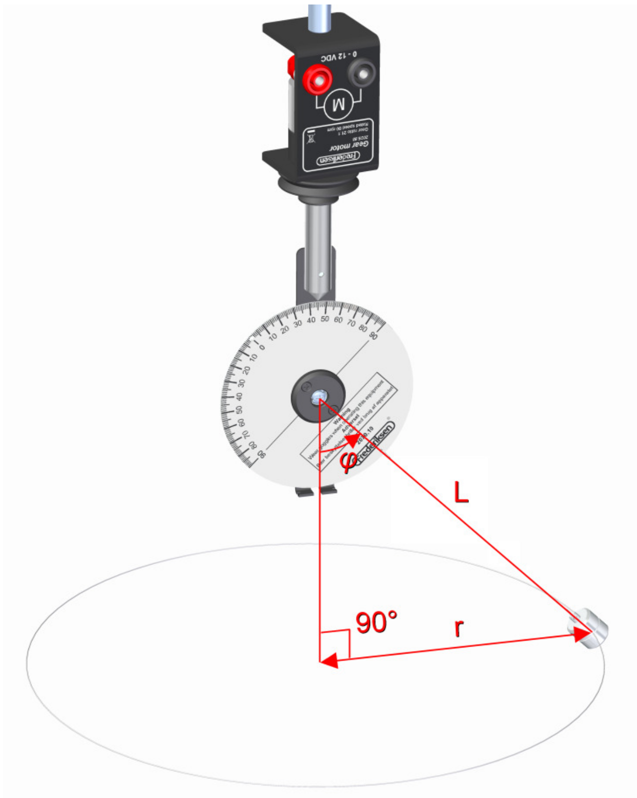
A stable setup can be made by e.g. two table clamps and three steel rods. If you use the front and rear edges of the table the pendulum bob doesn't have to move beyond the table top.

To use a photogate for period measurements, fasten a small (home-made) cardboard wing between the gear motor and the conical pendulum (see image page 1). On a SpeedGate, select *Period* and *Mean Period*.

With the motor at rest, place the string in the middle groove at the bottom of the holder; the graduated scale can now be used for adjusting the axle to a vertical position. Repeat the adjustment in two planes, perpendicular to each other.
When the axle is vertical, tighten all stand screws and lift the string to one side of the holder. It will now be able to turn the scale when the device rotates.

The motor runs on DC and needs a power supply that can provide a continuously adjustable voltage from 0 to 12 V. Normally, the current is less than 10 mA.

Start the motor by turning up slowly to allow the bob to follow the movements. Or else, the bob may hang straight down or even swing to the wrong side, preventing the scale from turning.



Procedure

The numbers in (parentheses) refer to the formulas in the theory section.

1 – The centripetal force and the orbital radius

Complete one or more series of measurement with a fixed orbital period and varying radius:

Weigh the bob and hang it up with the *longest* of the cords.

Measure the length L of the pendulum all the way from the centre of the scale to the middle of the bob.

Adjust the voltage for the motor to make the thread form an angle to the vertical of approx. 60°. Read the angle φ precisely – it takes a little practice.
Measure the rotational period T. If you use a stopwatch, take the average of for instance 10 turns.

Exchange the cord with one of the others and measure the new value for L.

Adjust the voltage for the motor in order to achieve the same orbital period as before. (This will be about the same as before.) If you don't use a photogate, you must fine-tune a bit at the time and measure the period carefully.
Read the new angle.

Repeat with the remaining cords.

This procedure can be repeated for other values of T by starting with the longest cord again and a slightly different angle.

With a SpeedGate: Remember to *reset* between measurements to obtain correct mean value.

2 – The centripetal force and the orbital period

Complete one or more series of measurement with a fixed radius and varying periods:

Weigh the bob and hang it up with the *shortest* of the cords.

Measure the length L of the pendulum all the way from the centre of the scale to the middle of the bob.

Adjust the voltage for the motor to make the thread form an angle to the vertical of approx. 60°. Read the angle φ precisely – it takes a little practice.
Measure the rotational period T. If you use a stopwatch, take the average of for instance 10 turns.

Use (5) to find the orbital radius r from the angle φ.

Exchange the cord with one of the others and measure the new value for L.
We want to keep the radius constant. Calculate the value of φ that gives the same radius as before. Use formula (5) again.)

Adjust the voltage to achieve the desired angle.
Measure the new period.

Repeat for the other cords.

This procedure can be repeated for other values of r by starting with the shortest cord again, using for instance the angle 50°.

With a SpeedGate: Remember to *reset* between measurements to obtain correct mean value.

Theory

For the uniform circular motion, the following relations apply. The mass of the bob is m.

Angular frequency ω and orbital period T:

$$\omega = \frac{2\pi}{T} \qquad (1)$$

Speed v and radius r:

$$v = \omega \cdot r = \frac{2\pi \cdot r}{T} \qquad (2)$$

Centripetal acceleration a:

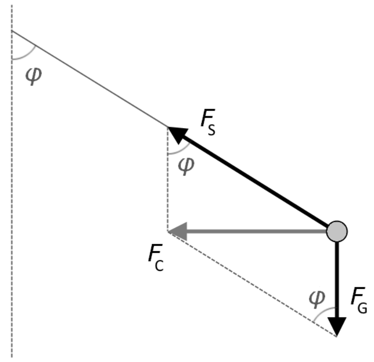
$$a = \omega^2 \cdot r = \frac{v^2}{r} = \frac{4\pi^2 \cdot r}{T^2} \qquad (3)$$

Centripetal force F_C:

$$F_C = m \cdot a = \frac{m \cdot v^2}{r} = \frac{4\pi^2 \cdot m \cdot r}{T^2} \qquad (4)$$

In a vertical plane, the centripetal force F_C is the result of the vector sum of the tension of the string F_S and the force of gravity F_G.

See figure.



The angle φ is read on the scale.

It is immediately noticed that with L as the length of the pendulum (measured from the centre of the scale to the centre of mass of the bob) we have:

The orbital radius r:

$$r = L \cdot \sin(\varphi) \qquad (5)$$

For the forces we have the following

Gravity F_G:

$$F_G = m \cdot g \qquad (6)$$

Centripetal force F_C and gravity F_G:

$$F_C = F_G \cdot \tan(\varphi) \qquad (7)$$

Calculations

Calculate the force of gravity F_G on the bob.

Below, you will try to demonstrate a theoretical relationship based on your measurements.

It is a good idea to plot the results in a suitable coordinate system.
If the theoretical relationship is *linear* the graphical method is a simple and very convincing tool.

If the directly measured quantities don't have a linear relationship, you can often compose some auxiliary expressions that have.

1 – The centripetal force and the orbital radius

Calculate using (5) the orbital radius r for each case. Next, use (7) to calculate the centripetal force F_C on the bob for each case.

Present the results in a suitable table.

Based on the theoretical formula for the centripetal force (4), and keeping T constant – which kind of relation do you expect between r and F_C?

Can this relationship be demonstrated from the experimental results?

2 – The centripetal force and the orbital period

Calculate the centripetal force on the bob F_C based on the gravity for each measured angle. Use (7).

Present the results in a suitable table.

Based on the theoretical formula for the centripetal force (4), and keeping r constant – which kind of relation do you expect between T and F_C?

Can this relationship be demonstrated from the experimental results?

Discussion and evaluation

With this experiment you try to give an experimental confirmations of a theoretical relation between certain physical quantities.

It would be a strange coincidence if the results should fit to the last decimal – measurement uncertainties will always be present.

Evaluate how well the theory describes the experiment.
Do the deviations look like random uncertainties or is there evidence of systematic errors?

Teacher's notes

Concepts used

Uniform circular motion
Centripetal force
Gravity
Decomposition of forces into components

Mathematical skills

Vectors or similar geometrical understanding of decomposition of forces.

Trigonometry

The students may need some help in order to find the entities to plot in part 2.

Didactic considerations

At least two different approaches can be used with this equipment. It will probably be a good idea to choose only one approach – or at least make students aware of when you change perspective:

a – Gravity is known, we examine the theory for uniform circular motion

Often, the students will know formula (6) in advance, and know how to calculate the force of gravity F_G based on the table value of g .

Then you use (7) – which simply expresses the decomposition of a vector (i.e. simple geometry) – for determining the left side of equation (4) which can then be examined.

This is the point of view of the experiments in this manual.

b – The theory for uniform circular motion is known, vi want to find the value of g .

By solving the system of equations (4) to (7) with respect to g we get

$$g = \frac{4\pi^2 \cdot r}{T^2 \cdot \tan(\varphi)} = \frac{4\pi^2 \cdot L \cdot \cos(\varphi)}{T^2} \qquad (8)$$

If (8) is to be used in an educational context, the previous relations should be reasonably well established for the students.

This approach may also lead to a number of considerations on assessing and minimizing the uncertainties of the corresponding quantities.

This is the point of view of the experiment 135730-EN Conical pendulum – determining g .

Detailed equipment list

Specifically for the experiment

207010 Conical pendulum
202550 Gear motor

Timing

Option: Timing with a SpeedGate:
197570 SpeedGate

Option: Timing with traditional photogate:
200250 Electronic counter
197550 Photocell unit

(If a photogate is *not* used:
148550 Digital stopwatch)

Standard lab equipment

DC power supply, like:
361600 Power supply 0-12V, 3A

Stand material, for example:
000800 Retort stand rod 150 cm
000810 Retort stand rod 100 cm (2 pcs.)
002310 Square Bosshead (3-4 pcs.)
001600 Table clamp (2 pcs.)

105750 Safety cables, silicone, 200 cm, black
105751 Safety cables, silicone, 200 cm, red