Conical pendulum – measuring *g*

Number	135730-EN	Topic	Mechanics, two-dimensional motion				
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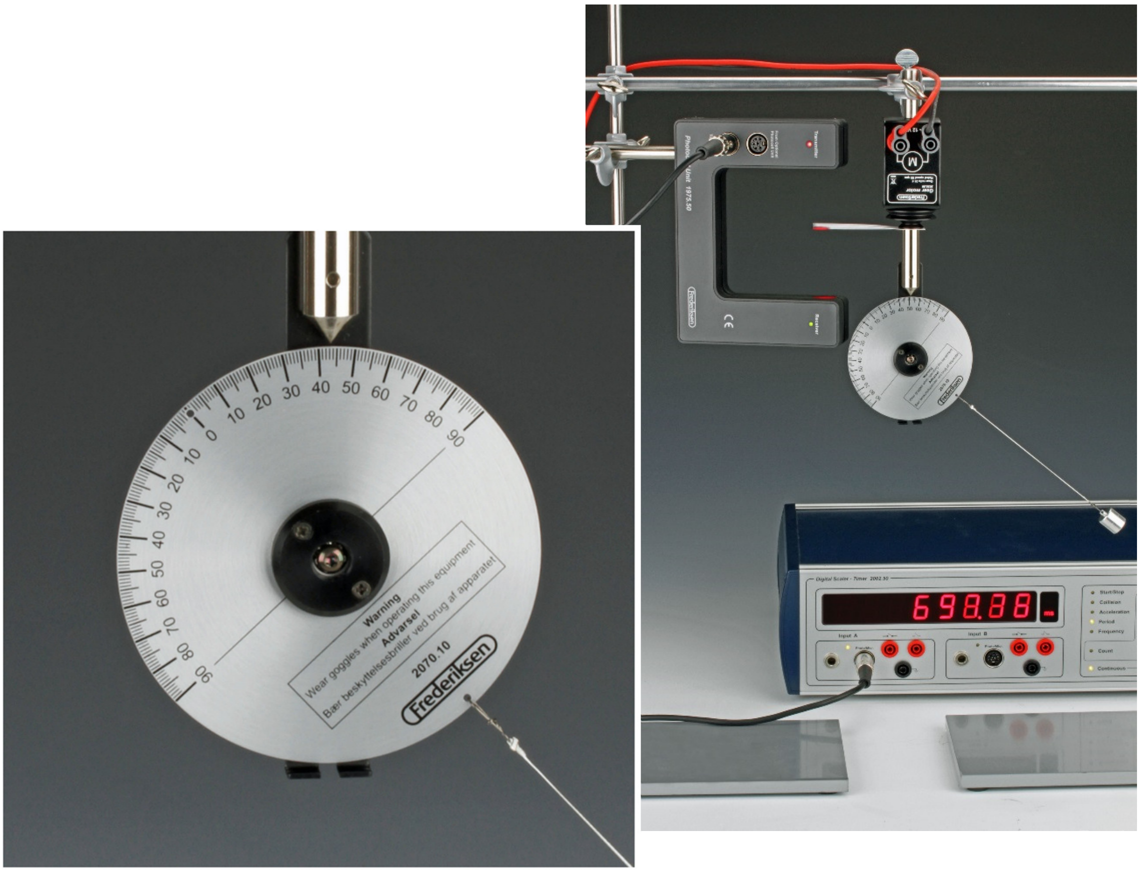


Objective

To determine the acceleration due to gravity by means of a conical pendulum.

Principle

We use a conical pendulum in this experiment. The bob performs a circular motion under the influence of the tension of the string and the force of gravity. The angle between these two forces is read on the fly on the graduated scale on the conical pendulum.

The orbital period can be found with a stopwatch or with a photogate.

From the measured quantities, *g* can be calculated.

Equipment

(See Detailed List of Equipment at the last page)

207010 Conical pendulum
202550 gear motor

DC power supply
Stand material

SpeedGate – *or*
Photogate and timer (– *or*
Stopwatch)

Setup

A stable setup can be made by e.g. two table clamps and three steel rods. If you use the front and rear edges of the table the pendulum bob doesn't have to move beyond the table top.

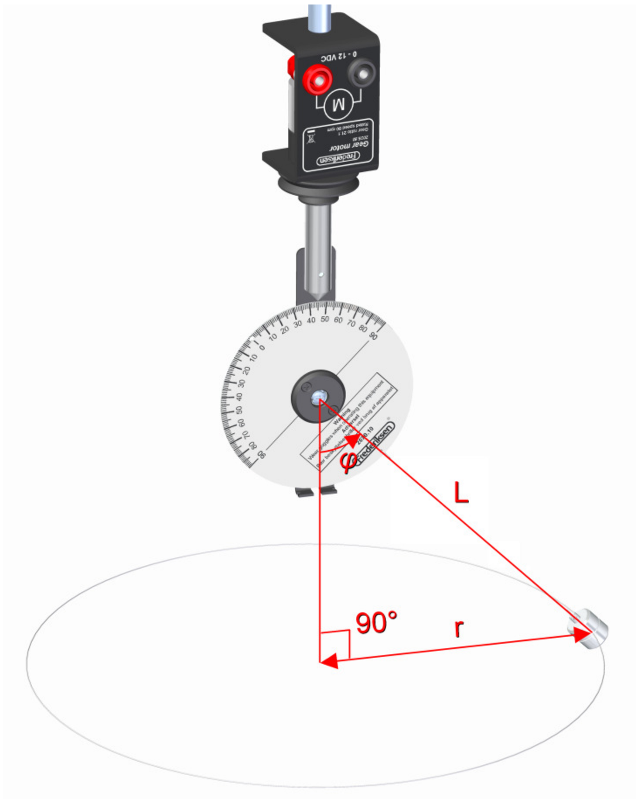
To use a photogate for period measurements, fasten a small cardboard wing between the gear motor and the conical pendulum (see image on page 1).

On a SpeedGate, select *Period* and *Mean Period*.

With the motor at rest, place the string in the middle groove at the bottom of the holder; the graduated scale can now be used for adjusting the axle to a vertical position. Repeat the adjustment in two planes, perpendicular to each other.
When the axle is vertical, tighten all stand screws and lift the string to one side of the holder. It will now be able to turn the scale when the device rotates.

The motor runs on DC and needs a power supply that can provide a continuously adjustable voltage from 0 to 12 V. Normally, the current is less than 10 mA.

Start the motor by turning up slowly to allow the bob to follow the movements. Or else, the bob may hang straight down or even swing to the wrong side, preventing the scale from turning.



Procedure

It is possible to isolate g in the system of equations that describes the conical pendulum – see eq. (8) in the theory section. This equation comprises three measured quantities:

Measure the length L of the pendulum all the way from the centre of the scale to the middle of the bob.

Adjust the voltage for the motor to make the thread form an angle to the vertical of approx. 60°. Read the angle φ precisely – it takes a little practice.

Measure the rotational period T. If you use a stopwatch, take the average for at least 10 turns.

With a SpeedGate, be sure to *Reset* between measurements to obtain correct mean value.

You have a number of cord lengths available, and you can vary the orbital period as you like. Try to set up a systematic measurement program before you start.

It is normally a good idea to keep one of the three measured quantities constant within a measurement series while varying another one systematically.

You must organize your time to ensure a good variation of the measured data.

Theory

For the uniform circular motion, the following relations apply. The mass of the bob is m .

Angular frequency ω and orbital period T :

$$\omega = \frac{2\pi}{T} \tag{1}$$

Speed v and radius r :

$$v = \omega \cdot r = \frac{2\pi \cdot r}{T} \tag{2}$$

Centripetal acceleration a :

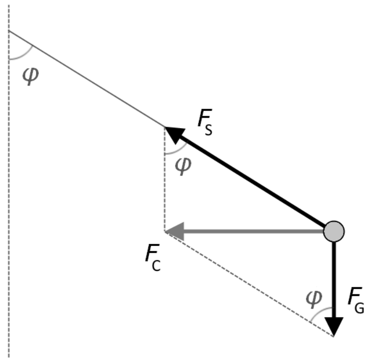
$$a = \omega^2 \cdot r = \frac{v^2}{r} = \frac{4\pi^2 \cdot r}{T^2} \tag{3}$$

Centripetal force F_C :

$$F_C = m \cdot a = \frac{m \cdot v^2}{r} = \frac{4\pi^2 \cdot m \cdot r}{T^2} \tag{4}$$

In a vertical plane, the centripetal force F_C is the result of the vector sum of the tension of the string F_S and the force of gravity F_G .

See figure.



The angle φ is read on the scale.

It is immediately noticed that with L as the length of the pendulum (measured from the centre of the scale to the centre of mass of the bob) we have:

The orbital radius r :

$$r = L \cdot \sin(\varphi) \tag{5}$$

For the forces we have the following

Gravity F_G :

$$F_G = m \cdot g \tag{6}$$

Centripetal force F_C and gravity F_G :

$$F_C = F_G \cdot \tan(\varphi) \tag{7}$$

By solving the system of equations (4) to (7) with respect to g we get

$$g = \frac{4\pi^2 \cdot r}{T^2 \cdot \tan(\varphi)} = \frac{4\pi^2 \cdot L \cdot \cos(\varphi)}{T^2} \tag{8}$$

Calculations

Present the measurements in a suitable table.

For each measured point (L, φ, T) you can of course calculate a value of g . The final result is found as the average of these.

Consider how accurately you can measure the quantities entering into the expression. Do some measuring situations give more precise results than others? Should all your results be weighted equally when calculating the average?

Consider other analysis options – e.g. plotting $4\pi^2 \cdot L \cdot \cos(\varphi)$ as a function of T^2 – for a measurement series with fixed L .

Discussion and evaluation

Compare your best bet for g with a table value.

Discuss whether there are systematic errors in the measurements, which may have affected the outcome.

Teacher's notes

Concepts used

Uniform circular motion
Centripetal force
Gravity
Decomposition of forces into components

Mathematical skills

Vectors or similar geometrical understanding of decomposition of forces.

Trigonometry

(Weighted average)

Didactic considerations

At least two different approaches can be used with this equipment. It will probably be a good idea to choose only one approach – or at least make students aware of when you change perspective:

a – Gravity is known, we examine the theory for uniform circular motion

Often, the students will know formula (6) in advance, and know how to calculate the force of gravity F_G based on the table value of g .

Then you use (7) – which simply expresses the decomposition of a vector (i.e. simple geometry) – for determining the left side of equation (4) which can then be examined.

This is the point of view of the experiments in the experiment 135710-EN Circular motion with conical pendulum.

b – The theory for uniform circular motion is known, vi want to find the value of g .

Equation (8) in the theory section is nothing but a solution to the system of equations (4) to (7) with respect to g .

If (8) is to be used in an educational context, the previous relations should be reasonably well established for the students.

This approach may also lead to a number of considerations on assessing and minimizing the uncertainties of the corresponding quantities.

This is the point of view of the present lab manual.

Detailed equipment list

Specifically for the experiment

207010 Conical pendulum
202550 Gear motor

Timing

Option: Timing with a SpeedGate:
197570 SpeedGate

Option: Timing with traditional photogate:
200250 Electronic counter
197550 Photocell unit

(If a photogate is *not* used:
148550 Digital stopwatch)

Standard lab equipment

DC power supply, like:
361600 Power supply 0-12 V, 3 A

Stand material, for example:
000800 Retort stand rod 150 cm
000810 Retort stand rod 100 cm (2 pcs.)
002310 Square Bosshead (3-4 pcs.)
001600 Table clamp (2 pcs.)

105750 Safety cables, silicone, 200 cm, black
105751 Safety cables, silicone, 200 cm, red