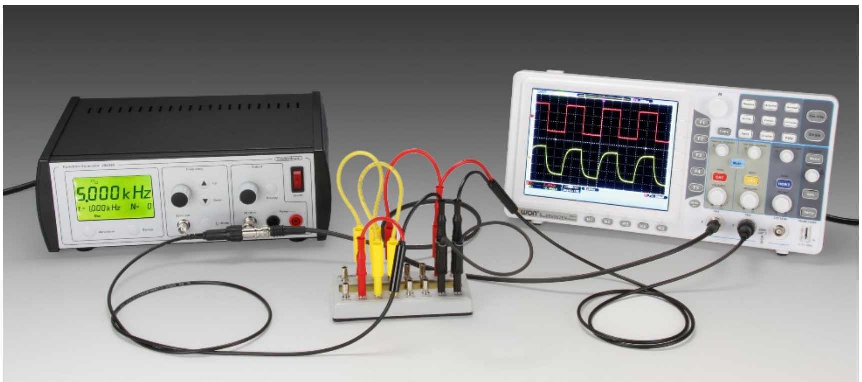
Frederiksen
Passion for science

Resonant circuits – measuring inductance

Number	136330-EN	*Topic*	Alternating current / electronics				
Version	2017-01-18 / HS	*Type*	Student exercise	*Suggested for*	grade 12+	*p.*	1/4



420600

Objective
By measuring resonance frequencies, coil inductance can be calculated. We work with two single coils, as well as series and parallel connections.

Principle
The (phase) resonance frequency is determined by using the oscilloscope in XY mode.

Equipment
LCR circuit 420600 includes the following components:

- Resistors:
  24.9 kΩ – 3.3 kΩ – 1.0 kΩ – 1.0 kΩ (1 %)
- Inductors:
  4.7 mH – 1.8 mH (5 %)
- Capacitors:
  2.2 nF – 1.0 nF (1 %)

The components are mounted with sockets that accommodates (shrouded and standard) banana plugs.

The resistors and the capacitors all tolerate more than 24 V DC or AC. The inductors tolerate up to 200 mA. None of these limits are exceeded in the circuits described in this manual.

Connections to function generator and oscilloscope are best made with two shielded cables (item no. 110002, BNC to safety plugs) – while the connection between function generator and oscilloscope is made with a BNC "T" and a standard BNC cable (110025).

As the shrouded plugs on cable 110002 aren't stackable, these cables must be connected last to the circuit.

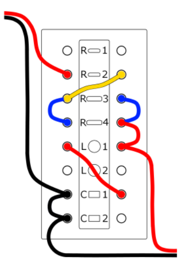
Now and then, you may need an extra socket for a ground lead – this can be fixed with an extra 25 cm safety cable, eventually placed in a socket to an unused component as shown below. (A component with only one leg connected isn't part of the circuit.)

The sketches in the manuals 136310 to 136350 all use the following colours:

Red:	Signal path
Black:	Ground
Blue:	Parallel connection of components
Yellow:	Serial connection of components

Input to the circuit is in all cases drawn to the left, the output is to the right.

In



Out

Measuring resonance frequency

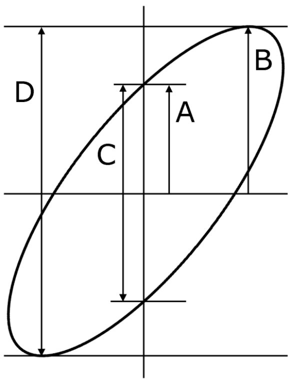
The input and output signals for a circuit with capacitors or inductors will not generally swing simultaneously (be in phase).

We will define the *resonant frequency* f_0 for the LCR resonant circuit (or filter) as the frequency at which the input voltage and input current are in phase. (More accurately this frequency is called the *phase resonance frequency*.)

In the circuit shown on the following page, u_1 and u_2 will have 0° phase difference at the frequency f_0.

(At f_0, the current is in phase with voltage u_1, and the voltage difference $u_1 - u_2$ is proportional with the current – hence, u_2 must be in phase with u_1.)

When the oscilloscope is in XY mode, and sinusoidal signals of the same frequency are applied to the inputs, the screen will show an ellipse – or possibly, as a special case, a line or a circle.
The phase difference φ between the two signals is determined by reading the distances C and D (or A and B) of the screen - see the figure below.
(Measuring A and B requires you to be careful to zero-adjust the y-signal.)

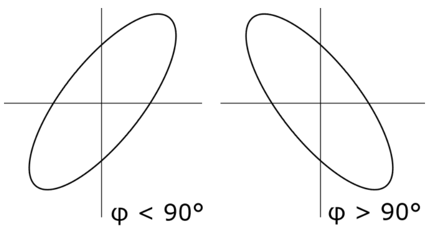


The following applies:

$$\sin(\varphi) = \frac{A}{B} = \frac{C}{D}$$

If the phase difference is larger than 90° the major axis of the ellipse will move from quadrant I and IV to quadrant II and III – see figure below.

For $\varphi = 90°$, the ellipse is symmetrical around the y axis.



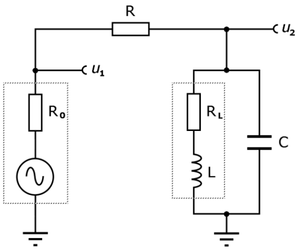
A phase difference of 0 or 180° will produce a straight line, sloping upwards, resp. downwards. (For larger phase differences than 180° you will see an ellipse again, and if you keep your head the value can still be found. We will not work with phase differences that large.)

As phase can be perceived as an angle you often hear the term "phase angle" for φ although this is a bit redundant.

Both signals are sine waves so on the 250350 function generator we will use the 50 Ω output.

Parallel resonance – measuring inductance

A resonant circuit, consisting of an inductor L (with internal resistance R_L) and a capacitor C, is fed as shown by a function generator through a resistor R. The 50 Ω output on the function generator is used.



With the components used here, you will find that maximum impedance and a phase difference of 0° occur at almost exactly the same frequency f_0, given by

$$f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{1}{L \cdot C}}$$

This frequency can be determined rather precisely. And as the capacitors used in 420600 has a 1 % tolerance, this formula can be used to find the precise value of L.

We will use C = 2.2 nF and R = 24.9 kΩ.

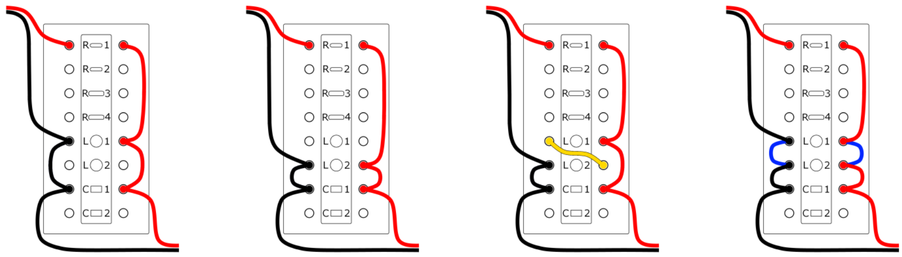
The output signal u_2 will be quite small when the frequency is far from f_0. But if you vary the frequency with a modest speed it is not difficult to find the interesting frequency interval to study.

Carry out the measurements and calculate the actual inductances for the inductors L_1 (nominal 4,7 mH) and L_2 (nominal 1,8 mH) – figure (a) resp. (b) below.

Likewise, determine the inductance of both the series and parallel connection of L_1 and L_2 – figure (c) resp. (d).

Compare the measured inductances for L_1 and L_2 with the nominal values.

Compare the inductances for the series resp. parallel connection with the theoretical values. Note: Use **the measured values** for L_1 and L_2 for this.



(a) (b) (c) (d)

Theory

The (phase) resonance frequency is by a very good approximation given by:

$$f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{1}{L \cdot C}}$$

Series connection of inductors:

$$L = L_1 + L_2$$

Parallel connection of inductors:

$$L = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}$$

Calculations

The very few calculations needed can be done with a calculator or math software.

Discussion and evaluation

Try to describe in words how the resonant circuit behaves. What is the significance of the resonance frequency?

Compare the nominal and the measured values of L_1 resp. L_2. Any deviations? Could this simply be explained by the tolerance (5 %) of the coils?

Compare theoretical and measured inductances for the series and parallel connections. Deviations? Do the component tolerances have any influence in this case?

Teacher's notes

Concepts used

Resonance frequency
Serial connection
Parallel connection
Resonant circuit

Mathematical skills

(Trigonometric functions)

About the equipment

The 1 kΩ resistors tolerate 1 W.
The other resistors: 0.6 W.
(These power limits will not be exceeded by using normal 0-24 V power supplies or our function generators.)

The capacitors tolerates at least 250 V.

The inductors (coils) has maximum RMS currents of 240 mA (4.7 mH) resp. 210mA (1.8 mH).
As the coils are wound on ferrite cores, a saturation phenomenon will be observed: The inductance drops when the current increases. To minimise this effect, keep signal levels low – never use peak currents larger than 200 mA.

Didactical considerations

Please be aware that many high school math text-books will define a separate concept called "phase shift" which **differs** from its normal use in university mathematics and in physics. It may be a good idea to have a look in your students' math books to avoid any confusion.
It should be safe to talk about "phase difference" or "phase angle" for the quantity φ used in the present text.

The Q-value for the resonant circuit is not mentioned in this text but it would be a natural extension to introduce it, if time permits.

This experiment can with advantage be combined with experiment 136340-EN LCR Band-pass and band-stop filters.

If you continue to 136350-EN LCR low-pass filters, please notice the two **different** values for the phase difference (measured with the oscilloscope) that indicates the resonance: In the present text, the voltage u_2 is measured over the complete resonant circuit – in 136350-EN it is measured over a capacitor.

Detailed equipment list

Specifically for this experiment

420600 LCR-circuit

Larger equipment

250350 (or 250250) Function generator

400150 Oscilloscope, digital 60 MHz
or
400100 Oscilloscope 60 MHz PC-USB

Standard lab equipment

110002	Cable, BNC to two safety plugs (2 ea.)
111100	BNC T adapter
110025	Coaxial cable w. BNC connectors, 50 Ohm
105710	Safety test lead 25cm, black
105711	Safety test lead 25cm, red (2 ea.)
105712	Safety test lead 25cm, yellow
105713	Safety test lead 25cm, blue (2 ea.)