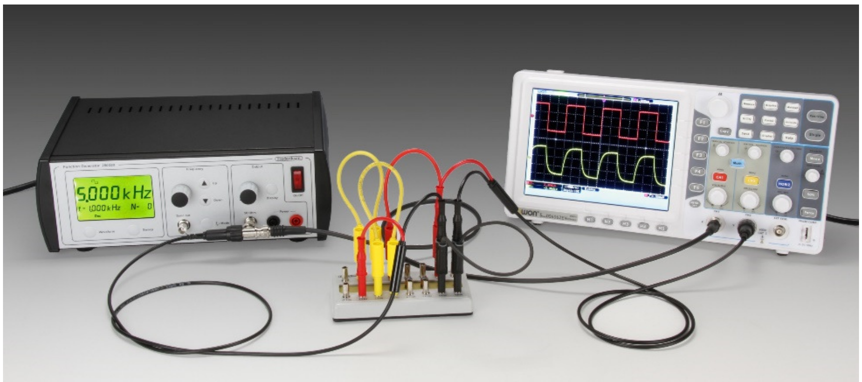
Frederiksen
Passion for science

LCR Low-pass filters

Number	136350-EN	*Topic*	Alternating current / electronics				
Version	2017-01-18 / HS	*Type*	Student exercise	*Suggested for*	grade 12+	*p.*	1/5



420600

Objective
The behaviour of LCR low-pass filters are studied by measuring the frequency response and step response.

Principle
The cut-off frequency (or phase resonance frequency) is determined by using the oscilloscope in XY mode.

Frequency response: The amplitude of a sine wave signal is measured before and after passing the filter. The ratio of the signal amplitudes are plotted in a logarithmic coordinate system.

Step response: A square wave signal is used as input and the output is studied on an oscilloscope.

Equipment
LCR circuit 420600 includes the following components:

- Resistors:
  24.9 kΩ – 3.3 kΩ – 1.0 kΩ – 1.0 kΩ (1 %)
- Inductors:
  4.7 mH – 1.8 mH (5 %)
- Capacitors:
  2.2 nF – 1.0 nF (1 %)

The components are mounted with sockets that accommodates (shrouded and standard) banana plugs.

The resistors and the capacitors all tolerate more than 24 V DC or AC. The inductors tolerate up to 200 mA. None of these limits are exceeded in the circuits described in this manual.

Connections to function generator and oscilloscope are best made with two shielded cables (item no. 110002, BNC to safety plugs) – while the connection between function generator and oscilloscope is made with a BNC “T” and a standard BNC cable (110025).

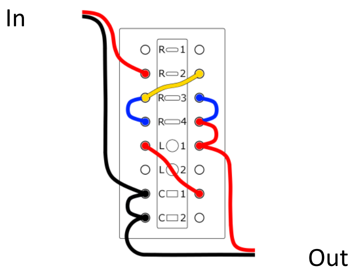
As the shrouded plugs on cable 110002 aren’t stackable, these cables must be connected last to the circuit.

Now and then, you may need an extra socket for a ground lead – this can be fixed with an extra 25 cm safety cable, eventually placed in a socket to an unused component as shown below. (A component with only one leg connected isn’t part of the circuit.)

The sketches in the manuals 136310 to 136350 all use the following colours:

Red:	Signal path
Black:	Ground
Blue:	Parallel connection of components
Yellow:	Serial connection of components

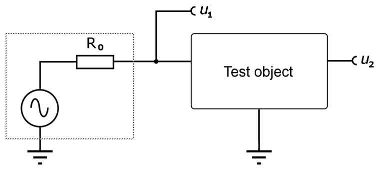
Input to the circuit is in all cases drawn to the left, the output is to the right.



Frederiksen Scientific A/S
Viaduktvej 35 · DK-6870 Ølgod
Tel. +45 7524 4966
Fax +45 7524 6282
info@frederiksen.eu
www.frederiksen.eu

Frequency response

The setup below shows the function generator hooked up to a test object, e.g. a filter.



The signals at the input and the output of the filter are designated u_1 resp. u_2.
The signal from the generator must be a sine wave.
The drawing shows explicitly that the generator has an output impedance, here $R_0 = 50\ \Omega$ as we are using the normal output.

The two voltages are monitored by an oscilloscope. With a modern digital oscilloscope, you can directly measure the size of the voltages. If this is not possible you must measure the peak-to-peak voltage on the screen. (There is no need for converting this into an RMS value – as long as you stick to the same type of voltage for all measurements.)

When both voltages u_1 and u_2 are measured as a function of the frequency f, the frequency response of the test object can be determined as

$$A(f) = \frac{u_2}{u_1}$$

It is important to notice that by considering the *ratio* between the voltages, any (frequency dependent) voltage drop over R_0 is irrelevant.

A is normally plotted in a double-logarithmic coordinate system (a log-log plot).

A *low-pass filter* is a circuit that allows signals with a frequency lower than a certain limit to pass, while more high frequency signals are damped.

A *high-pass filter* works exactly opposite.

Any real filter has a more or less soft transition; there cannot be a sharp cut off at a certain frequency.

The simplest filters consist of just a resistor (R) and a capacitor (C). It turns out that you can make filters with a sharper cut-off by also using an inductor (L) in the filter.

With the components available on the 420600 board many different LCR filters can be made. In this experiment we choose four combinations that represent four distinct behaviours.

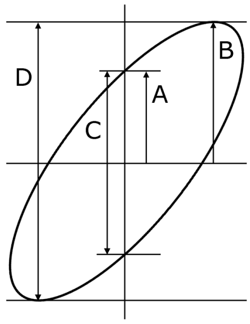
Measuring resonance frequency

We will define the *resonant frequency* f_0 for the LCR resonant circuit (or filter) as the frequency at which the input voltage and input current are in phase.
(A more accurate term is *phase resonance frequency*.)

In the circuits shown on the following page where u_2 is measured across the capacitor, u_1 and u_2 will have 90° phase difference at the frequency f_0.

(For a capacitor, the voltage is delayed 90° compared to the current.)

When the oscilloscope is in XY mode, and sinusoidal signals of the same frequency are applied to the inputs, the screen will show an ellipse – or possibly, as a special case, a line or a circle.
The phase difference φ between the two signals is determined by reading the distances C and D (or A and B) of the screen - see the figure below.
(Measuring A and B requires you to be careful to zero-adjust the y-signal.)

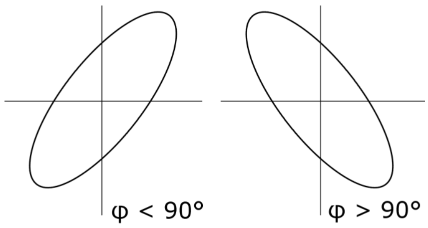


The following applies:

$$\sin(\varphi) = \frac{A}{B} = \frac{C}{D}$$

If the phase difference is larger than 90° the major axis of the ellipse will move from quadrant I and IV to quadrant II and III – see figure below.

At $\varphi = 90°$ the ellipse is symmetrical around the y axis.



A phase difference of 0 or 180° will produce a straight line, sloping upwards, resp. downwards. (Larger phase differences than 180° shows an ellipse again, and if you keep your head the value can still be found. We will not work with phase differences that large.)

As phase can be perceived as an angle you often hear the term “phase angle” for φ although this is a bit redundant.

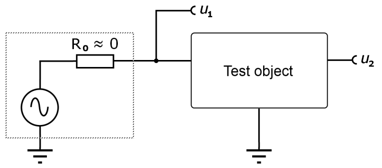
Both signals are sine waves so on the 250350 function generator we will use the 50 Ω output.

We will here define the *cut-off frequency* to be the phase resonance frequency.

Step response

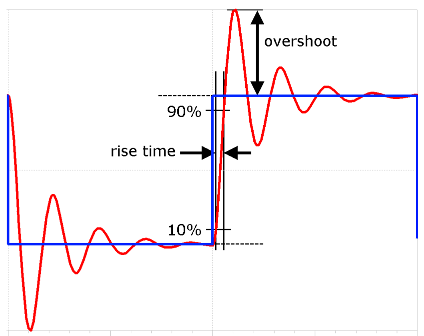
For this measurement, the power output on the function generator is used. Select square waves.

With the component values used here, a frequency of around 5 kHz is suitable. Hence we can profit of the negligible output impedance of the power output.



If you can make screen dumps from the oscilloscope, these can directly be used as measurement results – if not, you can take a photo or simply draw the image on checkered paper.

On the drawing below, the blue curve represents the input signal while the red curve represents a possible output.



We will concentrate on the two parameters rise time and overshoot:

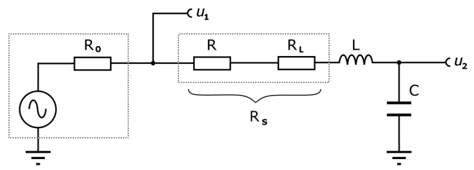
Rise time is the time it takes the signal to rise from 10 % to 90 % of the final step value.

Overshoot is the size of the eventual voltage peak above the final step value.

Sometimes a damped oscillation follows the voltage step. This is called *ringing*.

Be warned that while digital oscilloscopes often can measure rise time, the reading can be invalid in the case of ringing – check with the axis units and common sense.

1) LCR low-pass filter – frequency response



In this setup we have again the function generator with output impedance R_0.
The signal from the generator is fed to the input of the low-pass filter, consisting of a resistor, an inductor (i.e. a coil) and a capacitor.

Notice that both the outer resistor R and R_L – the internal resistance in the coil – are series connected with the ideal inductor L. The sum of these is R_S.
As we measure the actual value of u_1, R_0 is **not** considered part of the filter.

This circuit resembles the series resonant circuit – but now we pick the voltage across C as the output signal. The sharp resonance is damped by the resistor.

We will define a so-called *Q factor*, that turns out to characterize the behaviour of the filter. (This will be more pronounced in the following section.)

$$Q = 2\pi f_0 \cdot \frac{L}{R_S} = \frac{1}{R_S} \cdot \sqrt{\frac{L}{C}}$$

For fixed inductor and capacitor, Q can be tuned by selecting R (and thus R_S) appropriately. We will use L = 4.7 mH, C = 2.2 nF, and the values 500 Ω, 2000 Ω, 2914 Ω and 5300 Ω for R. (These are obtained through series and parallel connections.)

We have the following cases (their names will become clear later):

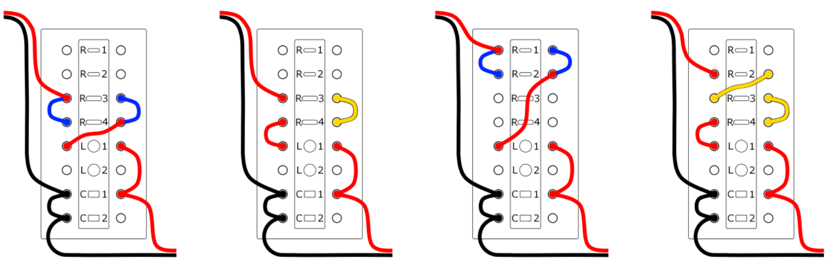
Underdamped:
R = 500 Ω R_S =509 Ω Q = 2.9

Flat response:
R = 2000 Ω R_S =2009 Ω Q = 0.73 $\approx \frac{1}{\sqrt{2}}$

Critical damping:
R = 2914 Ω R_S =2923 Ω Q =0.50 $\approx \frac{1}{2}$

Overdamped:
R = 5300 Ω R_S =5309 Ω Q = 0.28

(2914 Ω is a parallel connection of 24.9 kΩ and 3.3 kΩ. 5300 Ω is the series connection (1+1+3,3) kΩ.)



Underdamped   Flat response   Critical damping   Overdamped

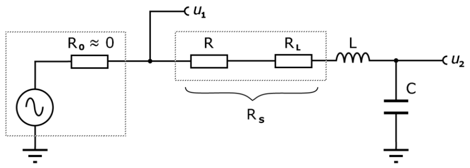
Complete the following measurement program for each of the four cases:

Determine f_0 where the phase difference is 90°.

Use the oscilloscope to measure the input voltage u_1 and the output voltage u_2 at f_0 as well as at the following frequencies:
5, 10, 20, 30, 40, 45, 50, 55, 60, 75, 100, 150, 200, 250, 300 kHz

Plot u_2/u_1 as a function of frequency. Use a logarithmic frequency axis. Comment the appearance.

2) LCR low-pass filter – step response



The setup resembles the previous; the only change is the use of the power output of the function generator where R_0 can be ignored. The signal from the generator is fed to the input of the filter. Use square waves and a frequency of 5 kHz.

Adjust the oscilloscope to make both signals u_1 and u_2 appear on the screen.

We will again examine the effect of the Q factor as defined above.

Use the same component values as before.

For each of the four cases the oscilloscope screen must be copied, photographed or drawn.
Measure in each case the *rise time* and the *overshoot*.
Comment – preferably in the context of the results from experiment 1.

Theory

The cut-off frequency (here takes as the phase resonance frequency) is with a very good approximation given by

$$f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{1}{L \cdot C}}$$

The Q factor is defined as

$$Q = 2\pi f_0 \cdot \frac{L}{R_S} = \frac{1}{R_S} \cdot \sqrt{\frac{L}{C}}$$

Frequency response for the LCR low-pass filter

$$A_{lowpass}(f) = \frac{1}{\sqrt{\left(1 - \left(\frac{f}{f_0}\right)^2\right)^2 + \frac{1}{Q^2} \cdot \left(\frac{f}{f_0}\right)^2}}$$

Presentation

Both the measured and the theoretical frequency response are most conveniently plotted using a spreadsheet.

Make both axes logarithmic. This will emphasize some points about the behaviour of the filters.

It is suggested to use the same coordinate system for all of the frequency response graphs.

Discussion and evaluation

Try to describe the behaviour of the four filters in words. What is the significance of the cut-off frequency and the Q factor?

What is the theoretical value of $A(f_0)$?

Comment on the names given for the four cases.

Compare theoretical and measured frequency responses.
Are there deviations? Can these simply be due to the tolerances (1-5 %) of the components?

Teacher's notes

Concepts used

Frequency response
Cut-off frequency
Serial connection
parallel connection

Step response
Rise Time
Overshoot

Mathematical skills

Logarithmic coordinate system
(Using a spreadsheet)

About the equipment

The 1 kΩ resistors tolerate 1 W.
The other resistors: 0.6 W.
(These power limits will not be exceeded by using normal 0-24 V power supplies or our function generators.)

The capacitors tolerates at least 250 V.

The inductors (coils) has maximum RMS currents of 240 mA (4.7 mH) resp. 210mA (1.8 mH).
As the coils are wound on ferrite cores, a saturation phenomenon will be observed: The inductance drops when the current increases. To minimise this effect, keep signal levels low – never use peak currents larger than 200 mA.

Didactical considerations

Decibel are not used (but can of course be introduced if you want to).

This is the reason that we don't mention explicitly the 12 dB / octave slope of 2nd order RCL filters.

Based on both the theoretical and the measured frequency responses, students should never the less be able to discover the existence of a fixed roll-off slope.

Please note that other definitions of cut-off frequency exist.

These measurements can for instance be combined with experiments 136310-EN RC Low-pass filters and 136320 RC High-pass filters.

In case you also work with 136330-EN Resonant circuits or 136340 LCR Band-pass and band-stop filters – please notice the two **different** values for the phase difference (as measured on the oscilloscope) that is used for indicating resonance: In this experiment the voltage u_2 is measured across the capacitor – in 136330-EN and 136340-EN, u_2 is measured across the complete resonant circuit.

Detailed equipment list

Specifically for this experiment

420600 LCR-circuit

Larger equipment

250350 (or 250250) Function generator

400150 Oscilloscope, digital 60 MHz
or
400100 Oscilloscope 60 MHz PC-USB

Standard lab equipment

110002	Cable, BNC to two safety plugs (2 ea.)
111100	BNC T adapter
110025	Coaxial cable w. BNC connectors, 50 Ohm
105710	Safety test lead 25cm, black
105711	Safety test lead 25cm, red (2 ea.)
105712	Safety test lead 25cm, yellow (2 ea.)
105713	Safety test lead 25cm, blue (2 ea.)